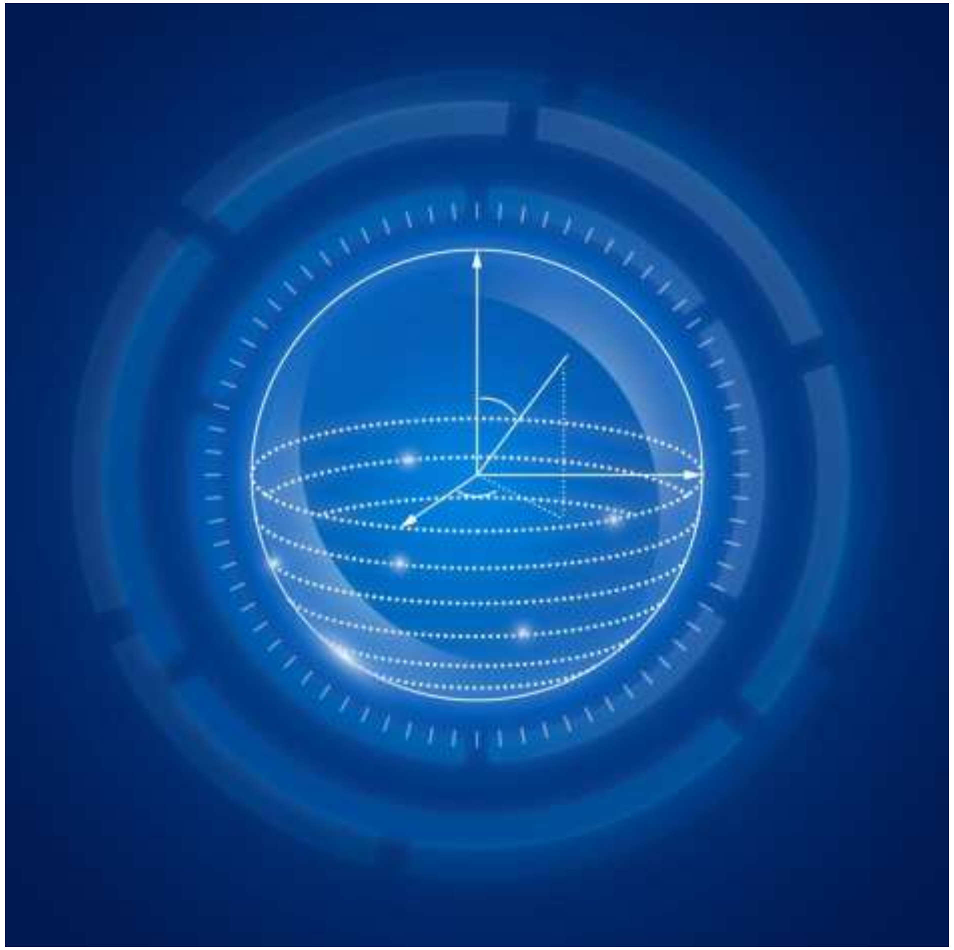


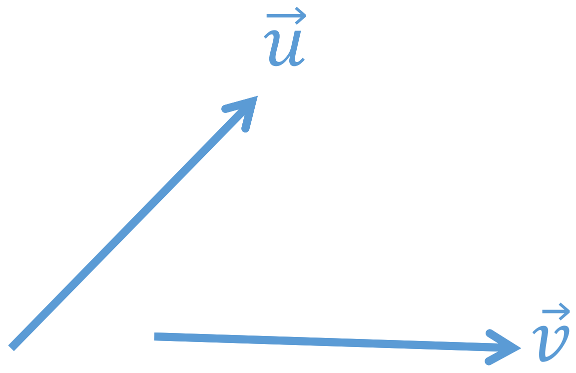
Scalar product and vector product

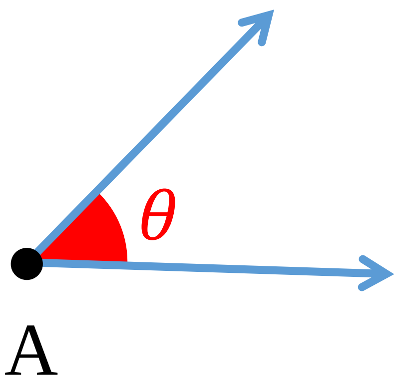
Scalar Product

Definition

$\vec{u}(x;y;z)$ and $\vec{v}(x';y'z')$ are two vectors in space.

- ✓ The scalar product of two vectors is a real number.
- ✓ Notation: $\vec{u}.\vec{v}$
- ✓ Rule: $\vec{u}.\vec{v} = \left|\left|\vec{u}\right|\right| \times \left|\left|\vec{v}\right|\right| \times \cos\theta$ where $\theta = (\widehat{\vec{u};\vec{v}})$
- ✓ Analytic form: $\vec{u}.\vec{v} = xx' + yy' + zz'$





Scalar Product

Properties

$\vec{u}, \vec{v}$ and $\vec{w}$ are three vectors in space.

❶ If $\vec{u} = \vec{0}$ or $\vec{v} = \vec{0}$, then $\vec{u}.\vec{v} = 0$

❷ $\vec{u}.\vec{v} = \vec{v}.\vec{u}$

❸ $(a\vec{u}).\vec{v} = \vec{u}.(a\vec{v}) = a(\vec{u}.\vec{v})$ where $a \in \mathbb{R}$.

❹ $\vec{u}.(\vec{v} + \vec{w}) = \vec{u}.\vec{v} + \vec{u}.\vec{w}$

❺ If $\vec{u}.\vec{v} = 0$ and $\vec{u} \neq \vec{0}$, $\vec{v} \neq \vec{0}$, then $(\widehat{\vec{u}, \vec{v}}) = \frac{\pi}{2}(2\pi)$

Scalar Product

Norm of a vector

$\vec{u}(x;y;z)$ and $\vec{v}(x';y'z')$ are two vectors in space.

❶ $\vec{u}^2 = ||\vec{u}||^2$

❷ $||\vec{u}|| = \sqrt{x^2+y^2+z^2}$

$\vec{u}^2 = \vec{u}.\vec{u} = ||\vec{u}|| \times ||\vec{u}|| \times \cos(\vec{u};\vec{u}) = ||\vec{u}||^2 \times \cos 0 = ||\vec{u}||^2$

$\vec{u}^2 = \vec{u}.\vec{u} = x \times x + y \times y + z \times z = x^2+y^2+z^2$

So $||\vec{u}|| = \sqrt{x^2+y^2+z^2}$

Scalar Product

Angle

$\vec{u}(x;y;z)$ and $\vec{v}(x';y'z')$ are two vectors in space.

$$\cos\theta = \cos(\vec{u};\vec{v}) = \frac{\vec{u}.\vec{v}}{||\vec{u}||\times||\vec{v}||} = \frac{xx'+yy'+zz\prime}{\sqrt{x^2+y^2+z^2}\times\sqrt{x'^2+y'^2+z'^2}}$$

Example:

$\vec{u}(2;3;1)$ & $\vec{v}(-1;2;3)$

$\vec{u}.\vec{v} = xx' + yy' + zz' = 2\times(-1) + 3\times 2 + 1\times 3 = -2 + 6 + 3 = 7$

$||\vec{u}|| = \sqrt{x^2+y^2+z^2} = \sqrt{2^2+3^2+1^2} = \sqrt{14}$

$||\vec{v}|| = \sqrt{x'^2+y'^2+z'^2} = \sqrt{(-1)^2+2^2+3^2} = \sqrt{14}$

$$\cos(\vec{u};\vec{v}) = \frac{\vec{u}.\vec{v}}{||\vec{u}||\times||\vec{v}||} = \frac{7}{\sqrt{14}\times\sqrt{14}} = \frac{7}{14} = 0.5$$

Scalar Product

Application of scalar product

❶ Scalar product is used to show that two vectors are orthogonal.

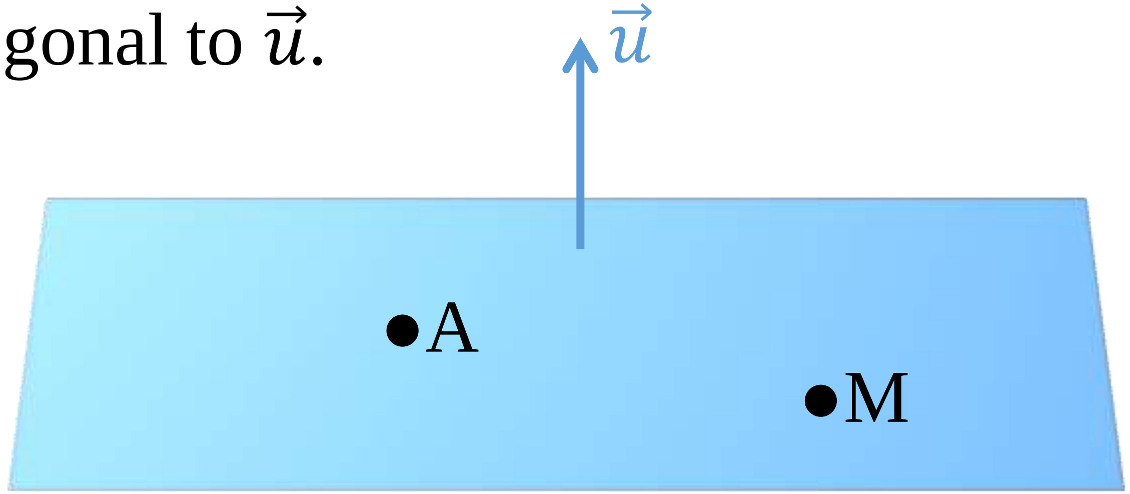
$\vec{u}.\vec{v} = 0$ and $\vec{u} \neq \vec{0}\,, \vec{v} \neq \vec{0}$

❷ level surfaces:

- The plane:

A is a fixed point, $\vec{u}$ is any vector.

The locus of a variable point M verifying $\vec{u}.\overrightarrow{AM} = 0$ is the plane that passes through A and orthogonal to $\vec{u}$.



Scalar Product

Application of scalar product

❶ Scalar product is used to show that two vectors are orthogonal.

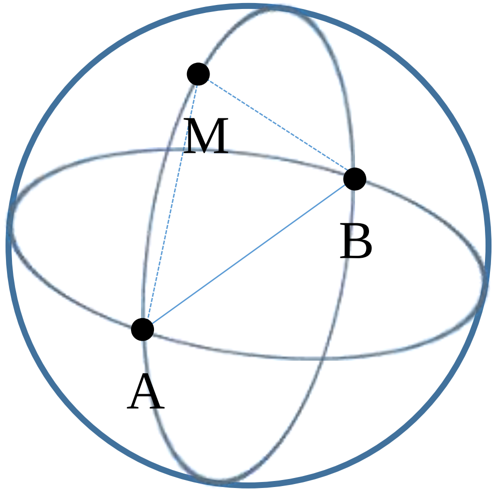
$\vec{u}.\vec{v} = 0$ and $\vec{u} \neq \vec{0}\,, \vec{v} \neq \vec{0}$

❷ level surfaces:

➢ The sphere:

A and B are two fixed points.

The locus of a point M in space verifying $\overrightarrow{AM}.\overrightarrow{BM} = 0$ is the sphere with diameter [AB].



Scalar Product

Application of scalar product

❸ Normal vector to a plane

$\vec{u}$ is a normal vector to a plane (ABC), if $\vec{u}$ is orthogonal to the two non collinear vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.